Names

What do the Axiom of Choice, the lambda calculus, de Bruijn indexes, verification of mobile systems, input, resource generation, automata with infinite alphabets, and several more yet-to-come theories have in common?

Names

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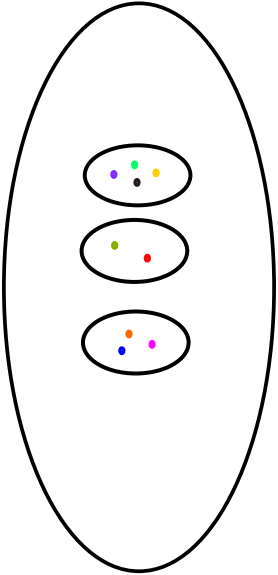
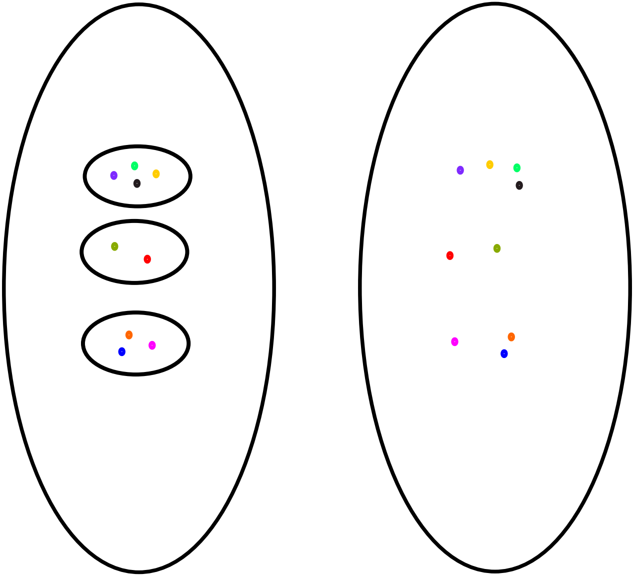
Pisa, October 10 2015

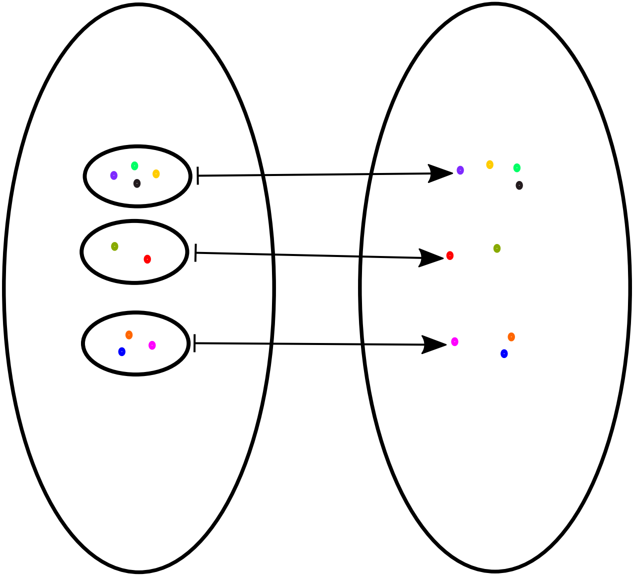
Independence of the Axiom of Choice

in three slides

(Fraenkel-Mostowski 1922-1938)







Permutations as constraints

$(X, \hat{\pi})$, π permutation of $\mathbb{N}$

$\hat{\pi} : X \to X$ permutation action

functions must be equivariant: $f(\hat{\pi}(x)) = \hat{\pi}(f(x))$

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Cumulative hierarchy, FM-sets, $\mathbb{N}$ urelements

Model of ZF but no C

Consider $(\mathbb{N}, \pi)$

Take the set of sets $\{\mathbb{N}\}$ containing just one set, $\mathbb{N}$

There is no choice function.

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Proof:

- observe $\pi(\mathbb{N}) = N$ and $\pi(n \in \mathbb{N}) \neq n$.
- $f(\mathbb{N}) = n \implies f(\pi(\mathbb{N})) = n \neq \pi(f(\mathbb{N}))$

Syntax with binders

In the permutation model

(Gabbay, Pitts 1999)

Abstract syntax: represent terms as trees

Nodes are operators, leaves are constants

Formally: initial algebras

Variable binding

"for all x in $A(x)$" "for all y in $A(y)$" "for all z in $A(z)$"

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infintely many indistinguishable variable names

$$
\begin{array}{lll}
\lambda x.t & \forall a.\phi & \texttt{let}\, k = E_1 \,\texttt{in}\, E_2 \\
\lambda y.t[y/x] & \forall b.\phi[b/a] & \texttt{let}\, h = E_1 \,\texttt{in}\, E_2[h/k] \\
\lambda z.t[z/x] & \forall c.\phi[c/a] & \texttt{let}\, s = E_1 \,\texttt{in}\, E_2[s/k]
\end{array}
$$

Abstract syntax with variable binding?

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Initial algebras in the permutation model

Abstract syntax with variable binding?

Initial algebras in the permutation model

Makes sense: no canonical choice

There is no interesting variable name

Urelements are names

Urelements are names

if $\pi(x) \neq x$ and $\hat{\pi}(t) \neq t$, then say x is a name of t.

If the names of t are finite, all the others may be freely interchanged without t being affected.

These are fresh names.

Semantics of mobile systems

(Plenty of excellent people, since around 1990)[1]

[1] and the speaker

The π-calculus

Semantics is an equivalence relation!

But it's nonstandard (fresh resources)

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Semantics is an equivalence relation!

But it's nonstandard (fresh resources)

... just like α-equivalence and binding is "non-standard" abstract syntax

Surprise!

[Montanari-Pistore 1996-2000]

Use the permutation model to give standard coalgebraic semantics to the pi-calculus.

Urelements are resources

Urelements are resources

Fresh names can be observed

generated, and then communicated
(appearing on labels of transitions)

Bisimulation up-to alpha-equivalence

Automata with infinite alphabets

From Kaminksi, Francez 1992, through several works

still ongoing[2]

[2] includes the speaker

Finite state automata accept finite words

Symbols come from finite alphabets

What if the alphabet is infinite?

Mobile systems, multi-user, security

Automata with memory registers

[Francez, Kaminski 1992]

the automaton can consume symbols in registers, or store new ones

Decidability of boolean operations

“Finite-memory automata”

Automata in the permutation model have been defined [e.g. Bojanczyk, Klin, Lasota 2011]

Equivalence with Francez-Kaminski [Ciancia, Tuosto, Tzevelekos - technical report]

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Urelements are symbols

"The language of words that start and end with the same symbol"

So many models of the same idea

So many papers to count[3]

[3]too many, if you count the speaker

Presheaf categories

model abstract syntax with binding as initial algebras

model name generation with final coalgebras

[Fiore, Moggi, Sangiorgi, Turi, Cattani, Winskel,... 1993-1999]

History dependent automata

states have registers

model name generation (in a finite way)
[Montanari, Pistore, 1996]

enjoy final, standard coalgebraic semantics
[Ciancia, Montanari 2012]

And what about De Bruijn indices? And what about this and that ...

What about urelements?

Enter category theory

[Fiore-Staton 2006]
Presheaves over finite sets, and HD-automata

[Gadducci, Miculan, Montanari 2006]
Also permutation model, G-sets ...

[Ciancia, Montanari, 2010]
Also De Bruijn indexes, with the proper choice of a Kan extension

Categorical equivalence

The morphisms matter more than the objects

Bidirectional translations

Categorical equivalence

The morphisms matter more than the objects

Bidirectional translations

up-to isomorphism

Urelements are the basic building block

[in presheaves] colimit completion

[in register automata & C] contents of registers

[in the permutation model] observables of elements

Nominal computation, and beyond

Nominal computation: study the theory of computation in variants of the permutation model

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$$P \neq NP$$

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(but it's not the key point...)

Can we change the building blocks and retain the building?

Graphs [Montanari, Sammartino 2014]

Equivalence relations [Bonchi, Buscemi, Ciancia, Gadducci 2012]

Partial orders [Bruni, Montanari, Sammartino 2015]

Reminder:

The axiom of choice still does not hold!

thank you for listening