UNIVERSITÀ DEGLI STUDI
DI MILANO

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Algorithmic in the 12th Century: the *Carmen de Algorismo* by Alexander de Villa Dei

HaPoC 2015 - Pisa

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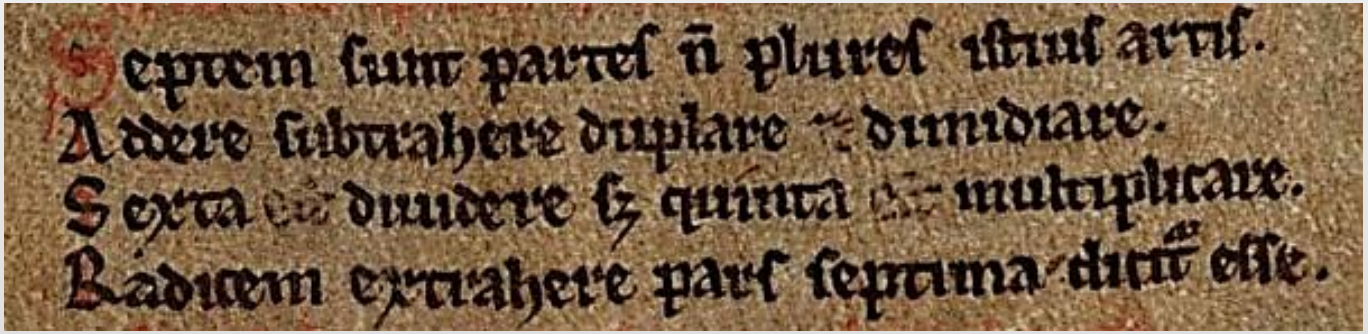
Algorismus

Hec algorismus ars praesens dicitur esse

What is an *Algorismus*?

As nearly everybody knows, the name is the latinization of “al-Khawarizmi”, a Chorazmian scientist who lived in the IX century and worked in Baghdad

In medieval Europe, *algorismus* means written work, devoted to the description of the 7 operations with Indo-Arabic numerals.



Carmen de Algorismo, or…

- ALGORISMUS:
  - Algorismus, Algorismus in integris, Algorismus in metro, Algorismus in versu, Algorismus integrorum, Algorismus metricus, Algorismus metrificatus, Algorismus versificatus, Algorithmus in metro, Ars algorismi, De algorismo
- POEM:
  - Carmen de algorismo, Carmen de algorismo seu arithmetica, Carmen de algorismus, Carmen de algorithmo, Carmen de arithmetica, Carmen de arte algorismi seu arithmetica, Metrical arithmetic, Metricus algorismus, Versus de Algorismo, The arithmetical poem, Lectura algorismi metrici, Poem on the Algorismus,
- HANDBOOK:
  - Libellus de algorismo, Regule algorismi, Treatise on arithmetic
- INDIAN ORIGIN: Indorum ars numerandi

The content of the «Carmen de Algorismo»

- Latin hexameters: 285 vs. 333
- Description of the «bis quinque Indorum figurae», of their numerical meaning and of positional notation;
- The 7 operations list, followed by their detailed descriptions:
  - Addition (with proof)
  - Subtraction (with proof)
  - Doubling and halving
  - Multiplication (with proof)
  - Division (with proof)
  - Progression (only in Steele)
  - Square and cube root extraction
- Operations are performed with non negative integers

Halliwell - Steele

Transcriptions

Halliwell (1841)

- British mss.

CARMEN DE ALGORISMO.

Hæc algorismus ars præsens dicitur;[2] in qua
Talibus Indorum[3] fruimur bis quinque figuris.
0. 9. 8. 7. 6. 5. 4. 3. 2. 1.
Primaque significat unum: duo vero secunda:
Tertia significat tria: sic procede sinistra
Donec ad extremam venias, quæ cifra vocatur;
Quæ nil significat; dat significare sequenti.
Quælibet illarum si primo limite ponas,
Simpliciter se significat: si vero secundo,

Steele (1922)

- British Library, Royal 8.C.iv, with additions from
  - Egerton 2622 (XV c.)
  - Royal 12.E.1 (XV c.)

Carmen de Algorismo.

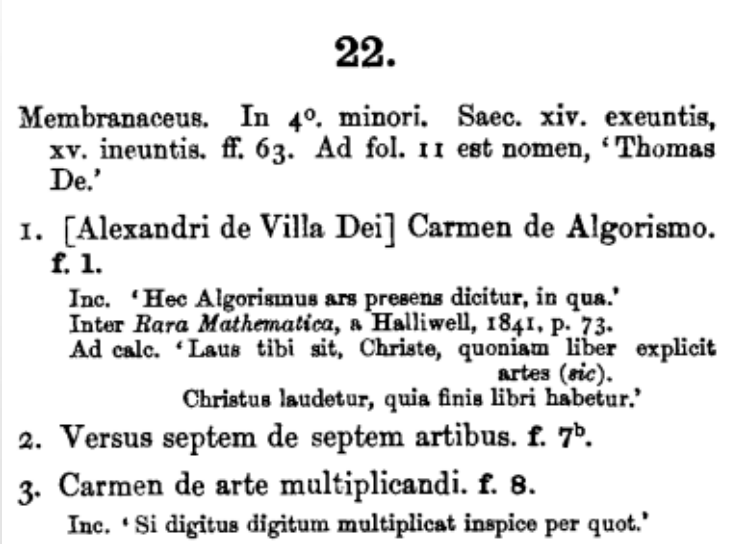
[*From a B.M. MS.*, 8 *C.* iv., *with additions from* 12 *E.* 1 *& Eg.* 2622.]

Hec algorismus ars presens dicitur[1]; in qua
Talibus Indorum[2] fruimur bis quinque figuris.
0. 9. 8. 7. 6. 5. 4. 3. 2. 1.
Prima significat unum: duo vero secunda:
Tercia significat tria: sic procede sinistre
Donec ad extremam venies, qua cifra vocatur;
[3][Que nil significat; dat significare sequenti.]
Quelibet illarum si primo limite ponas,
Simpliciter se significat: si vero secundo,
Se decies: sursum procedas multiplicando.[4]
[Namque figura sequens quevis signat decies plus,
Ipsa locata loco quam significet pereunte:
Nam precedentes plus ultima significabit.]

Addendum

- In some mss* dating XIII or XIV c., after the explicit of the Carmen text in Halliwell's version, 35 more verses appear.
  - Incipit: «Si digitus digitum multiplicat adspice per quot»
  - Explicit: «a maiore minus et summa videbitur eius»

In Oxford, Bodleian Library, Digby 22, these lines are entitled *Carmen de arte multiplicandi*;
it appears as a different work, as a third work separates it from AdV Carmen.



22.

Membranaceus. In 4°. minori. Saec. xiv. exeuntis, xv. ineuntis. ff. 63. Ad fol. 11 est nomen, 'Thomas De.'

1. [Alexandri de Villa Dei] Carmen de Algorismo. **f. 1.**
   Inc. 'Hec Algorismus ars presens dicitur, in qua.'
   Inter *Rara Mathematica*, a Halliwell, 1841, p. 73.
   Ad calc. 'Laus tibi sit, Christe, quoniam liber explicit artes (*sic*).
   Christus laudetur, quia finis libri habetur.'
2. Versus septem de septem artibus. **f. 7b.**
3. Carmen de arte multiplicandi. **f. 8.**
   Inc. 'Si digitus digitum multiplicat inspice per quot.'

* such as Pal. Lat. 1393; Erlangen, Universitätsbibliothek, 394

Commentary

- Oxford, ms. Digby 81, ff. 11-35
- Thomas de Novo Mercato Commentum in carmen <Alexandri de Villa Dei> de algorismo
- Algorismus metricus cum notis marginalibus (Praha)
- Algorismus in metro cum commento
- Royal 12 F XIX 183- Commentarium
- In British Library 8 C. IV it is accompanied by a prose interpretation found also in Sloane 513 (which gives the author's name 'secundum Saxton'), Egerton 851 and Add. 17716.

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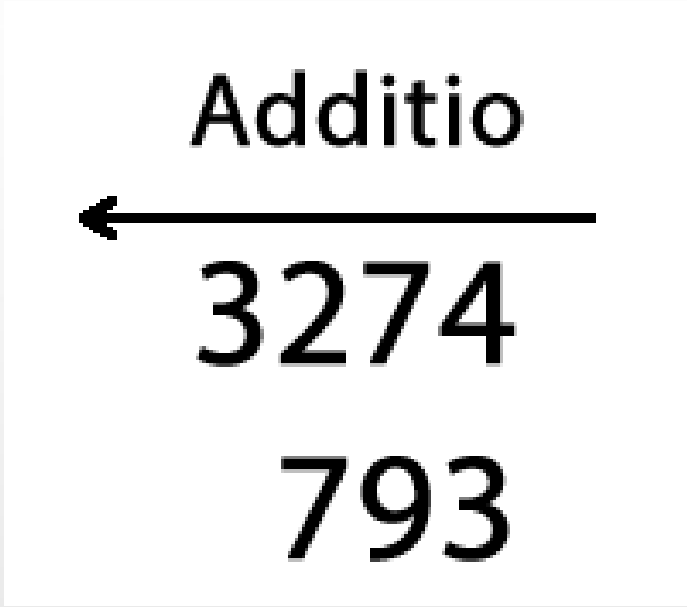
Calculation Techniques

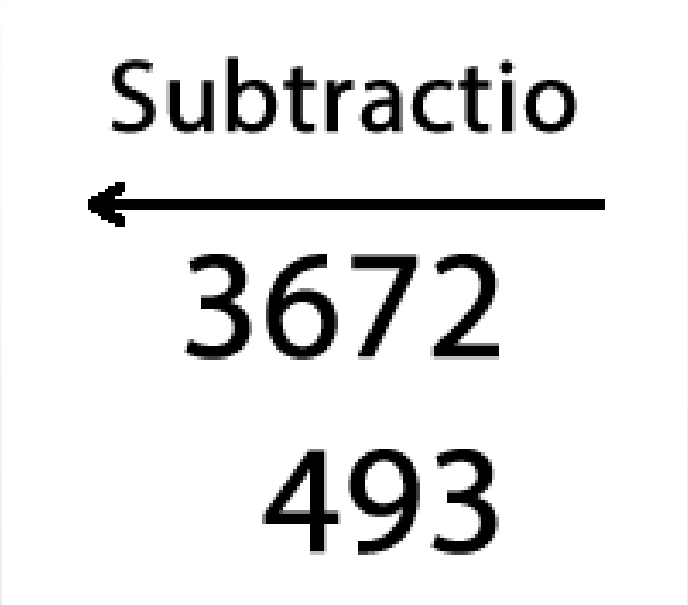
Addition

- It is performed from right to left on a sand table.
- Possible carries are registered above the first addend.
- The result replaces the first addend so that, once the sum has been completed, one can immediately check the correctness of the calculation by performing a subtraction.



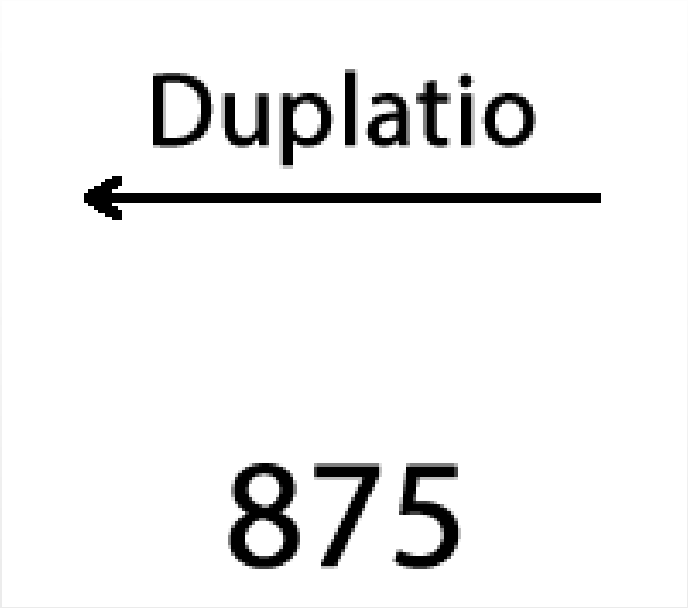
Subtraction

- It is performed from right to left on a sand table.
- Possible carries are registered above the minuend.
- The difference replaces the minuend so that, once the subtraction has been completed, one can immediately check the correctness of the calculation by performing a sum.



Doubling

- It is performed (on a sand table) on a single number from right to left by doubling the single digits and summing partial results.
- Possible carries are registered above the number.
- The result replaces the number so that, once the doubling has been completed, one can immediately check the correctness of the calculation by halving.

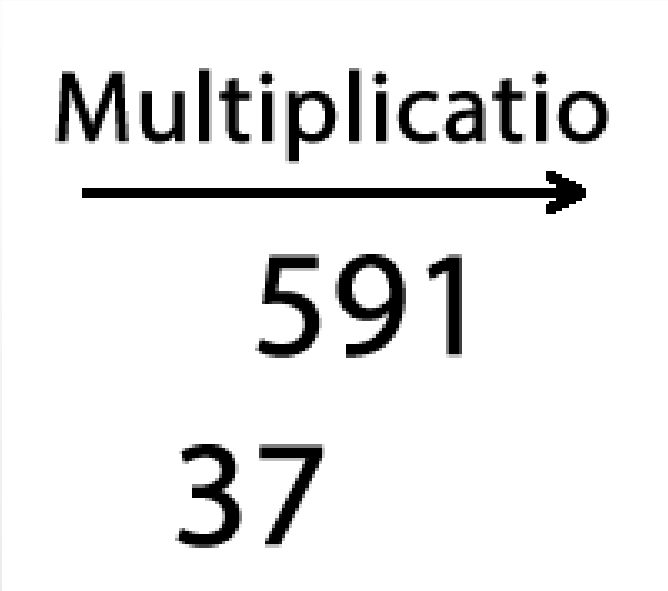


Halving

- It is performed (on a sand table) on a single number from left to right by halving the single digits.
- Possible partial remainders are registered above the number.
- The difference replaces the minuend so that, once the halving has been completed, one can immediately check the correctness of the calculation by doubling.

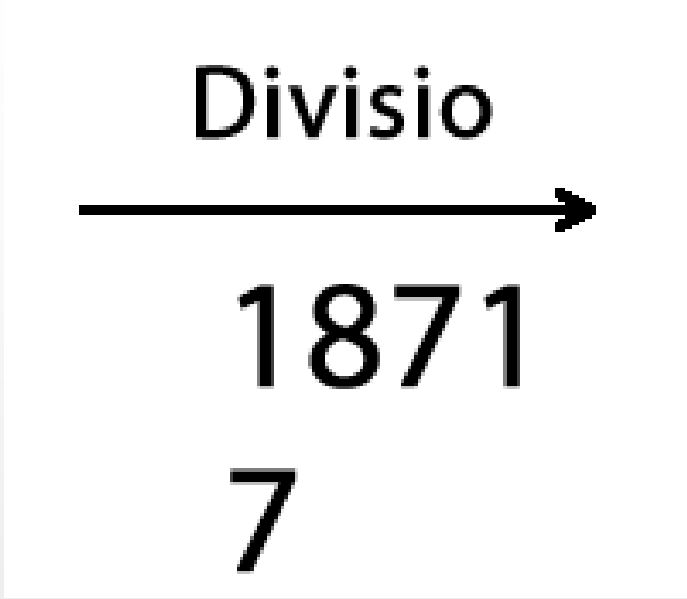
Multiplication

- It is performed from right to left on a sand table.
- Possible carries are registered above the first addend.
- The product replaces the first factor so that, once the multiplication has been completed, one can immediately check the correctness of the calculation by dividing.



Division

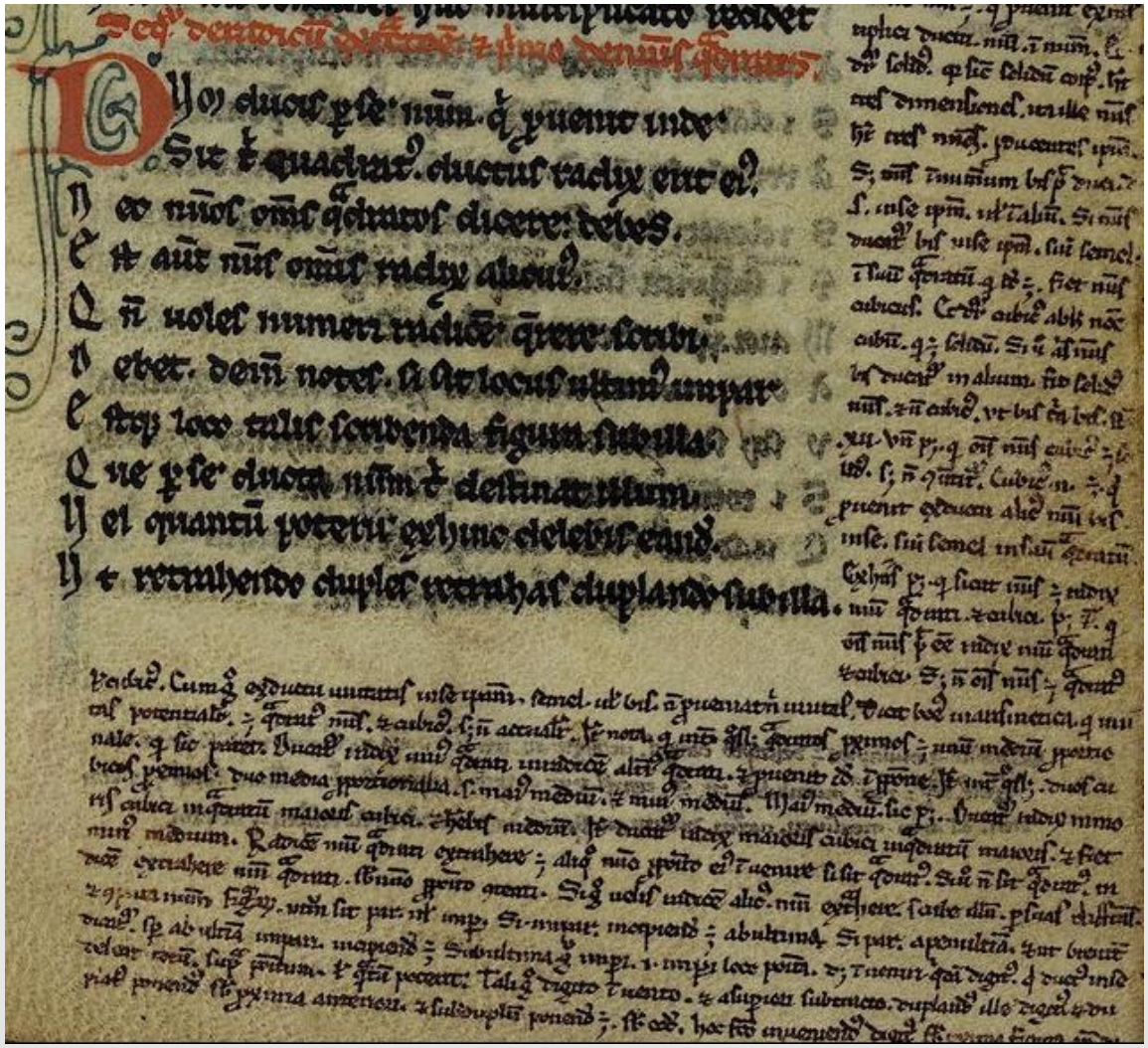
- It is performed from left to right on a sand table.
- Possible partial remainders are registered above the dividend.
- The quotient is written on the top so that, once the division has been completed, one can immediately check the correctness of the calculation by multiplying and then adding the possible remainder.



Square and Cubic Root Extraction

- It is performed from left to right on a sand table.
- Partial result is registered under the number.
- Information about how to write down single steps of calculation is quite generic and no numerical example is given.
- Description of the steps is so concise that it required (as in Pal. Lat. 1393) long glosses to explain the passages.
- The Carmen verses are literally surrounded by prose
- «algorismus ab inventore s(cilicet) ab algo quod est inductio et rismus quod est numerus»

BAV, Pal. Lat. 1393, ff. 61v-69r (13th cent.)



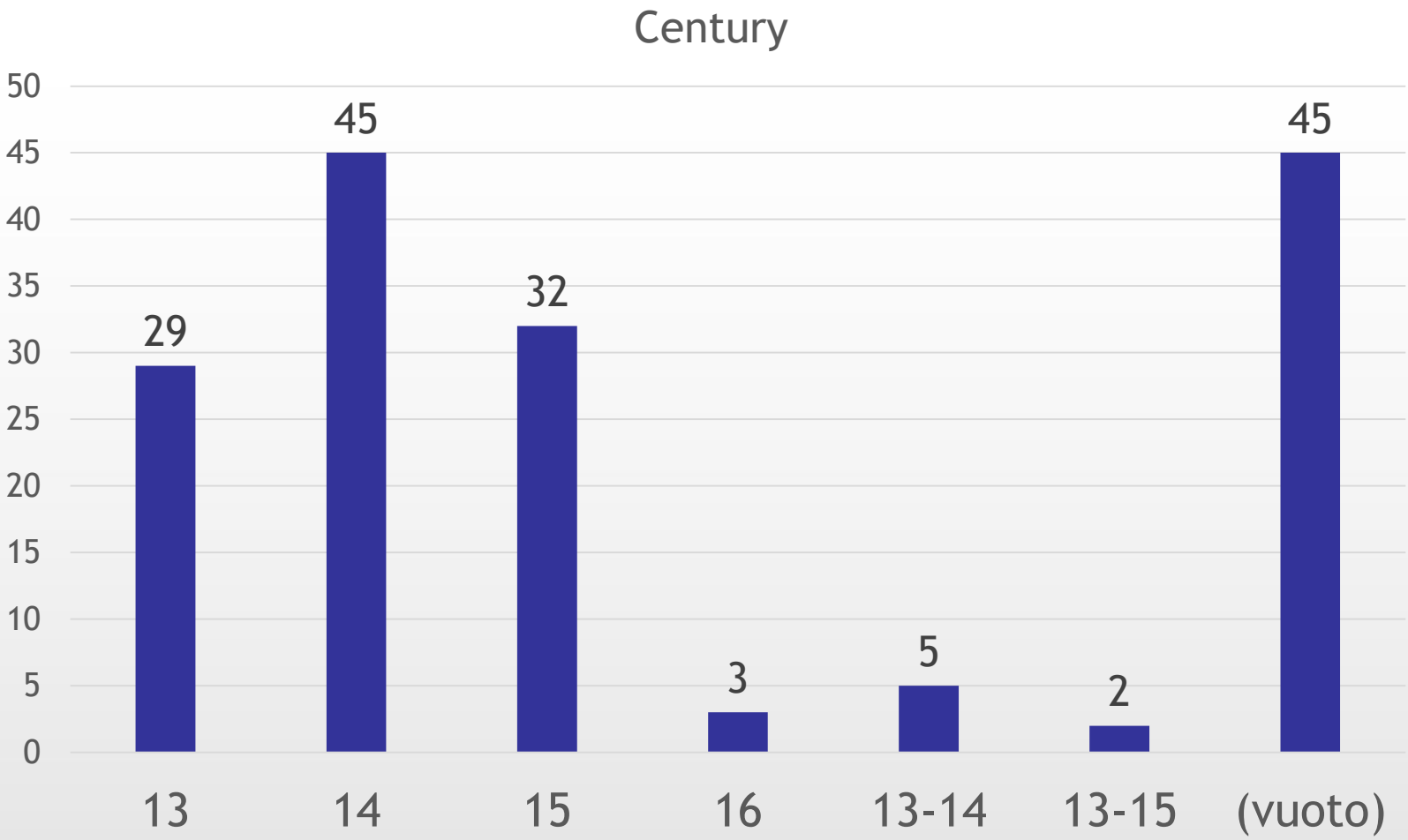
f. 65 r.

Spread

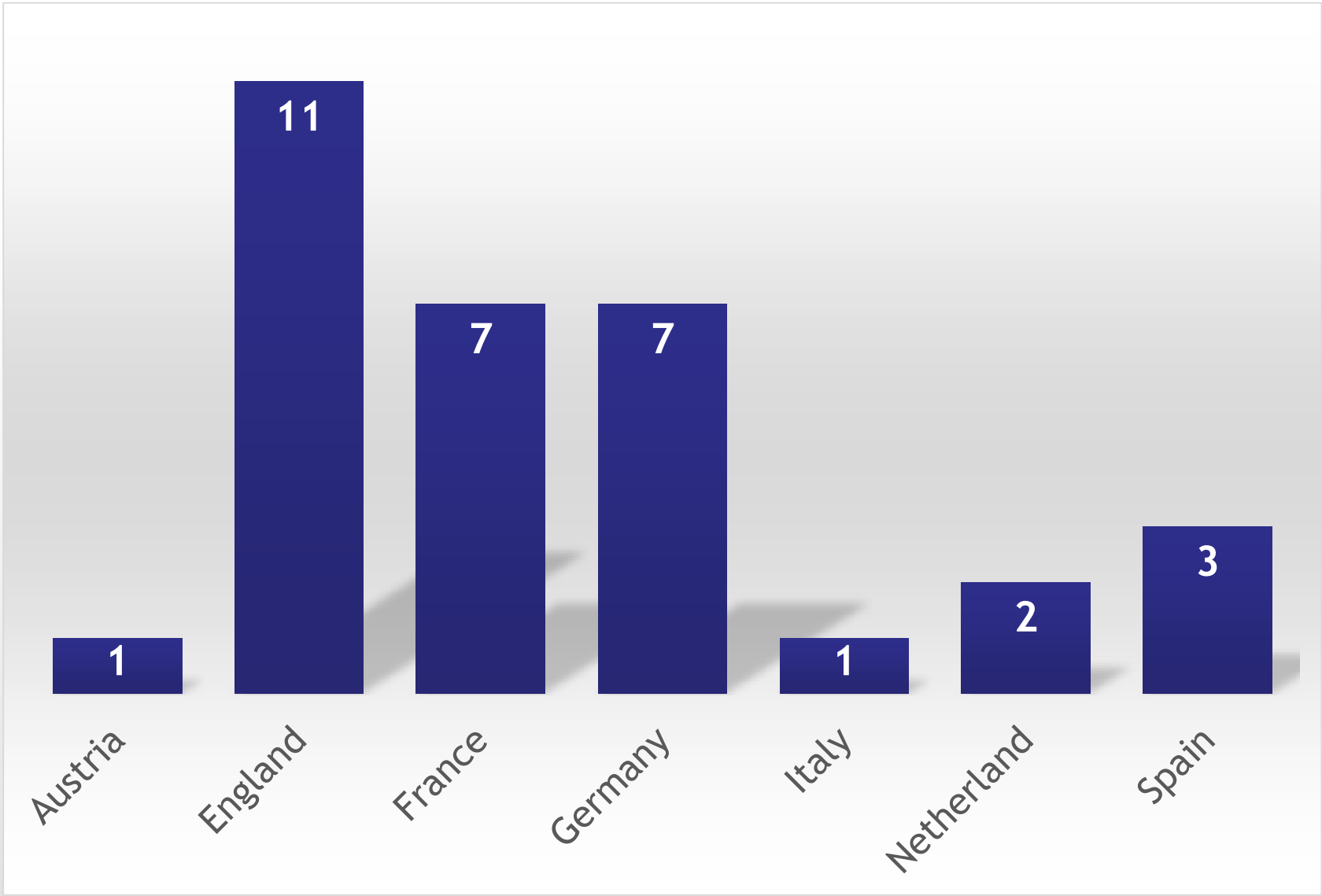
How many?

- Mss. with the following incipits have been considered:
  - Hec Algorismus ars presens dicitur
  - Haec Algorismus ars presens dicitur
  - Hic Algorismus ars presens dicitur
- Mss. catalogued as Carmen de Algorismo, Algorismus metricus (etc) and attributed to Alexandre de Villedieu (even without incipit)
- We added to our study, Assisi, Fondo Antico presso la Biblioteca del Sacro Convento, ms. 174
- 161

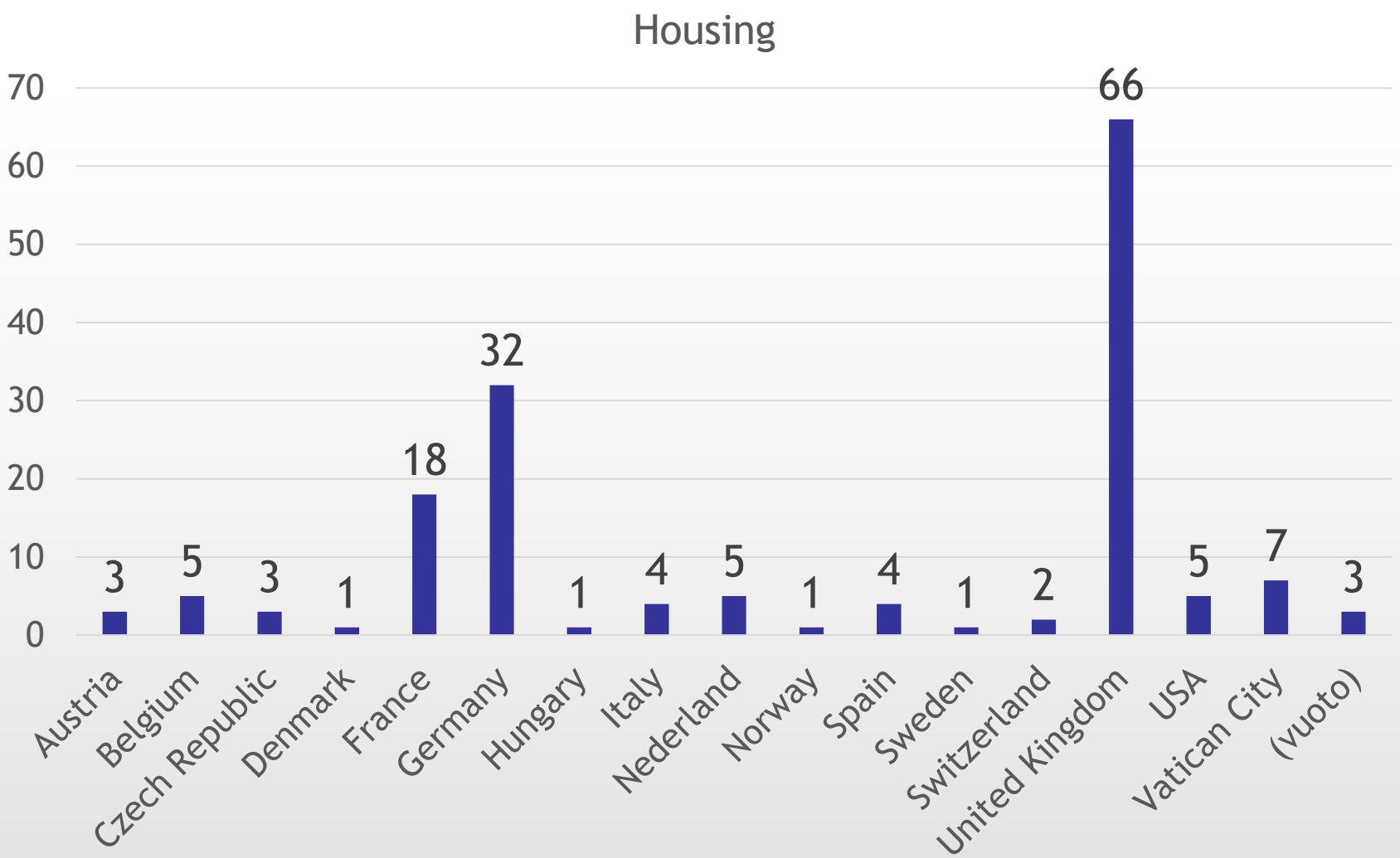
When?



Where are some of the mss. copied?



Where are they now?



Comparison with al-K

Kitāb al-Jam‘ wat-Tafrīq bi-Ḥisāb al-Hind

- Book of Addition and Subtraction According to the Hindu Calculation: the original work in Arabic is lost, but some Latin partial translations survive:
  - Dixit Algorizmi (DA)
  - Liber Ysagogarum Alchorismi (LY)
  - Liber Alchorismi (LA)
  - Liber Pulueris (LP)

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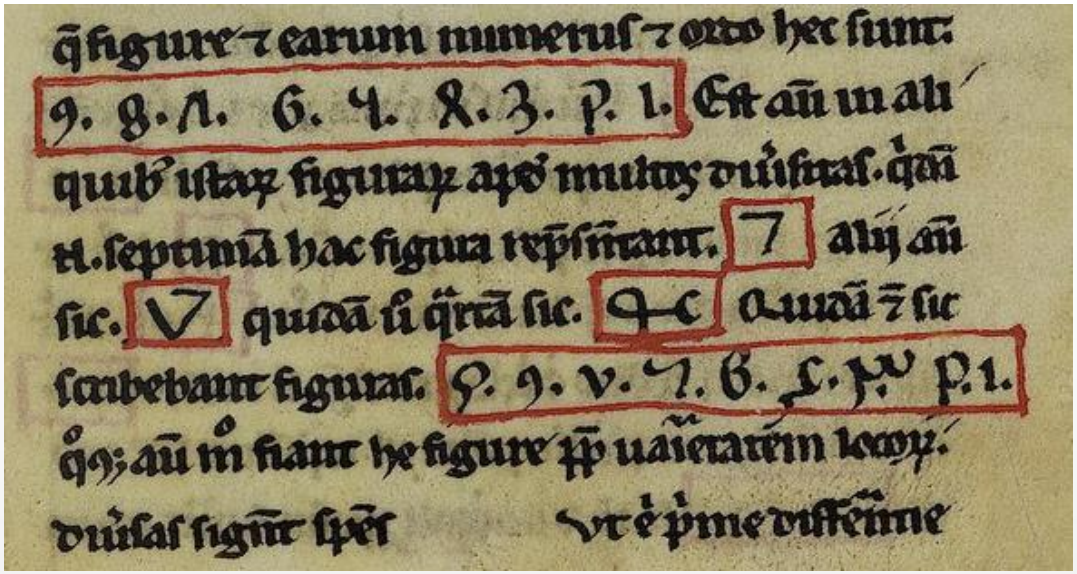
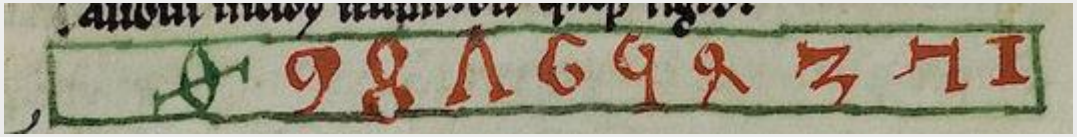
Numbering

Source	definition	representation
DA	Fecerunt (Yndi) IX literas, quarum figurae sunt he 987654321…	987654321
LY	his VIIII figuris 987654321 tam integros quam minutias significantibus utuntur.	987654321
LP/LA	Que figure et earum numerus et ordo est	987654321 ghubar
CdeA	Talibus Indorum fruimur bis quinque figuris	

LA vs CdeA

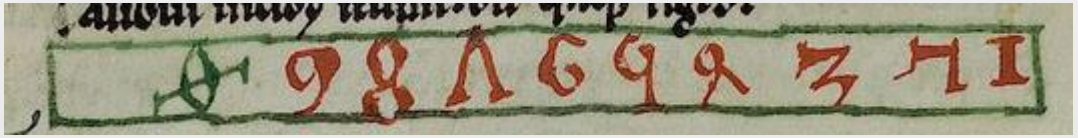




Biblioteca Apostolica Vaticana, Pal. lat. 1393

Cifra

Source	definition	representation
DA	(Yndi) posuerunt circulum paruulum in similitudine O litere	0
LY	Utuntur etiam ciffre	0 or τ
LP/LA	Circulus	-
CdeA	cifra vocatur; *Quae nil significat; dat significare sequenti.*	0 ø



BAV, Pal. lat. 1393

Cambridge, University Library, O.2.45

A Comparison

Operations Order	CdeA	DA	LY	LA/LP
$+$	1	1	2	1
$-$	2	2	3	2
$\times\, \mathbf{2}$	3	4	5	3
$\div\, \mathbf{2}$	4	3	4	4
$\times$	5	5	1	5
$\div$	6	6	6	6
$\sqrt{}$	7	?	7	7
$\sqrt[3]{}$	8	?	-	-

Base	CdeA	DA	LY	LA/LP
10	$\mathbb{N}_0$	$\mathbb{N}_0, \mathbb{Q}_0^+$	$\mathbb{N}_0, \mathbb{Q}_0^+$	$\mathbb{N}_0, \mathbb{Q}_0^+$
60	-	$\mathbb{N}_0, \mathbb{Q}_0^+$	$\mathbb{N}_0, \mathbb{Q}_0^+$	$\mathbb{N}_0, \mathbb{Q}_0^+$

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New calculation techniques

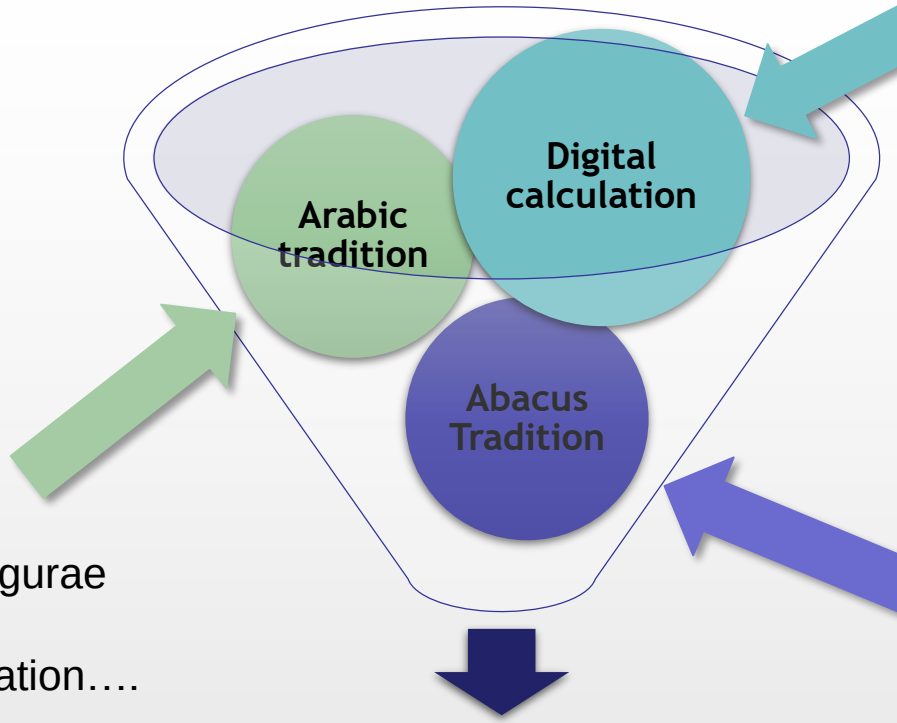
Curriculum studiorum in Paris in the 12th c.

- Grammar: Alexander de Villedieu's *Doctrinale* replaces Donatus' and Priscian's works
- New arithmetic
  - No abacus needed
  - Sand
  - Reduced role of memory
- Surviving contrast between speculative and practical arithmetic (logistics)

Quadrivium

- Study of the new arithmetic was not encouraged by the Sorbonne authorities, as even Roger Bacon refers:
  - Studium Parisiense adhuc non habuit usum istarum quinque scientiarum (Foreign languages, maths, perspective, experimental science, alchemy)
- Probably professors taught this contents outside the university.

Various traditions



- Integers
- Digiti, articuli, compositi
- Boetian tradition (monas)

- Indian source
- Bis quinque figurae
- chifra
- positional notation….

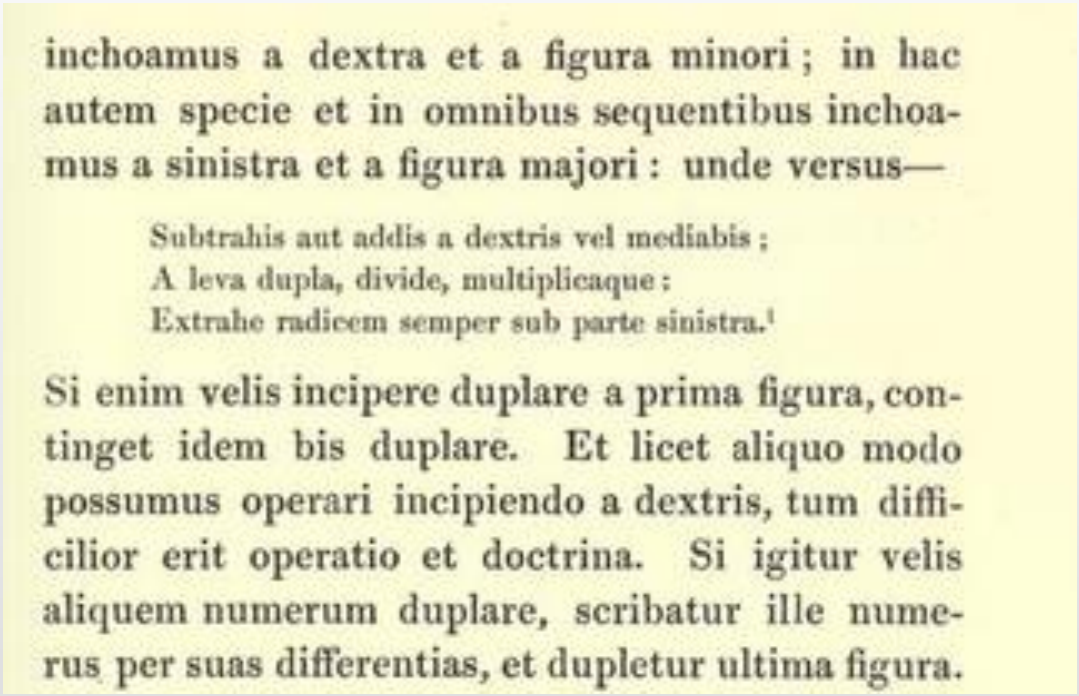
- Ghubar numerals
- Replacing operands
- Multiplication per differentias

Carmen de Algorismo

The Carmen Influence

Latin: Sacrobosco

- Sacrobosco was educated in Oxford, and he was a Paris master from 1221 until his death in 1244 or 1256
- His *Algorismus* becomes soon a popular handbook, due to its clarity
- Sacrobosco quotes the lines of the Carmen about operation verse; no reference to the author.



inchoamus a dextra et a figura minori; in hac autem specie et in omnibus sequentibus inchoamus a sinistra et a figura majori: unde versus—

Subtrahis aut addis a dextris vel mediabis;
A leva dupla, divide, multiplicaque:
Extrahe radicem semper sub parte sinistra.[1]

Si enim velis incipere duplare a prima figura, continget idem bis duplare. Et licet aliquo modo possumus operari incipiendo a dextris, tum difficilior erit operatio et doctrina. Si igitur velis aliquem numerum duplare, scribatur ille numerus per suas differentias, et dupletur ultima figura.

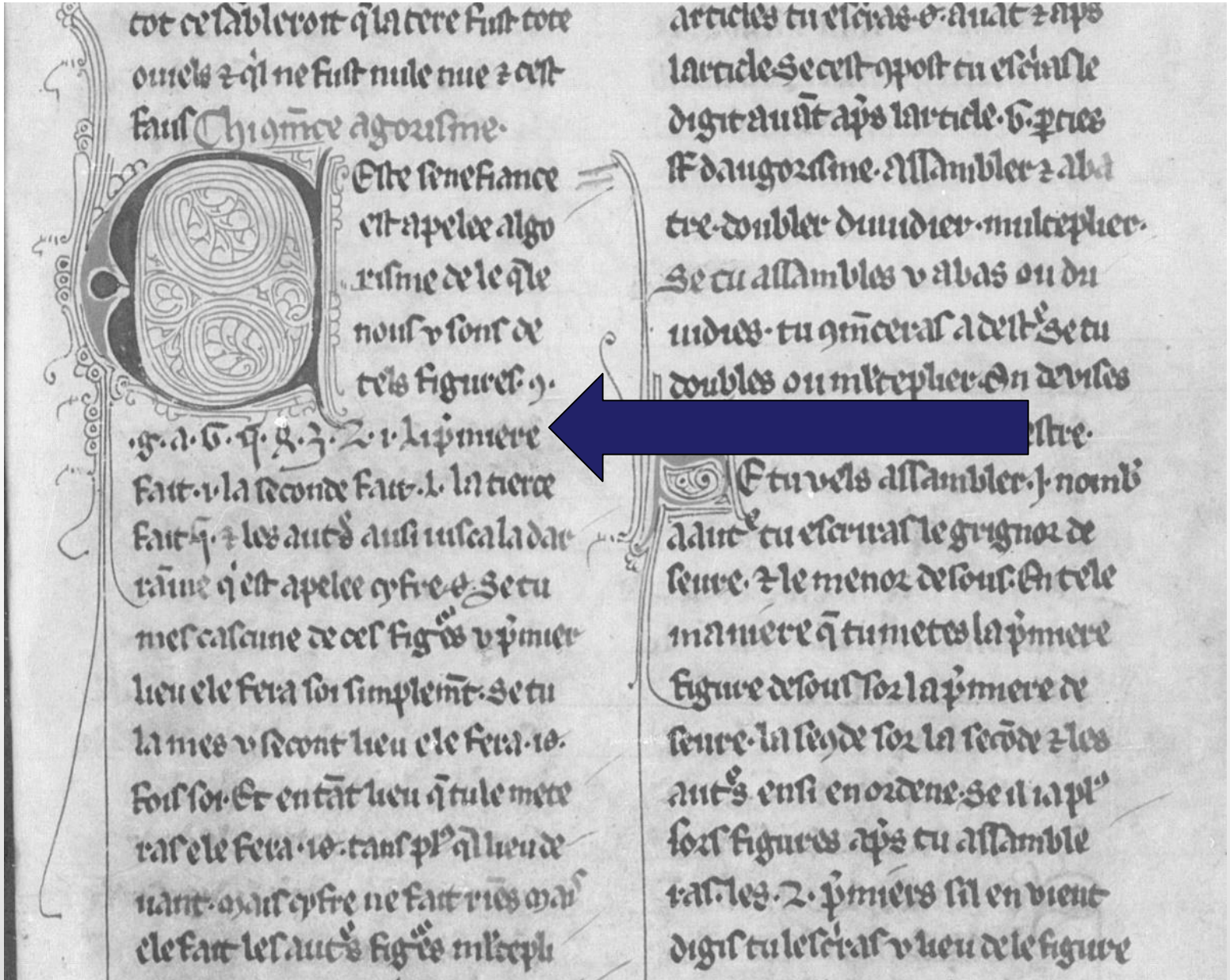
Algorismus secundum usum Cantabrigiensem

- The Carmen appeared difficult, especially if compared with the homonymous work in prose by Sacrobosco.
- Some scribes (scholars) began “interleaving” the two works so that some lines by Villedieu appeared commented by the corresponding passage by Sacrobosco.

Arithmetic in Vernacular

- Paris, Bibl. Sainte-Geneviève, 2200 f. 150r (XIII c.)
- Paris, BNF, ms. Français, Anc. 7929, 154-155
  - 6 parties sont d'augorisme: assambler et abatre, doubler, dimidier, multeplier, [deviser]. Se tu assambles u abas ou dimidies, tu commenceras a destre; se tu doubles ou multeplie[s] ou devises, tu commenceras a senestre.
- North of France appeared more connected with abacus tradition.
- In the South of France, in Montpellier, at the beginning of the XIV c. Florentine masters taught mathematics (Jacopo da Firenze e Paolo Gherardi).

Paris, BNF, ms. Français, Anc. 7929, 154-155



English Translation

- The Craft of Nombrynge (ms. Egerton 2622 - XV c.)
  - Text followed by an explanation in old English

[1] HEc algorismus ars presens dicitur; in qua
Talibus indorum fruimur bis quinque figuris.

[1] leaf 156 a.

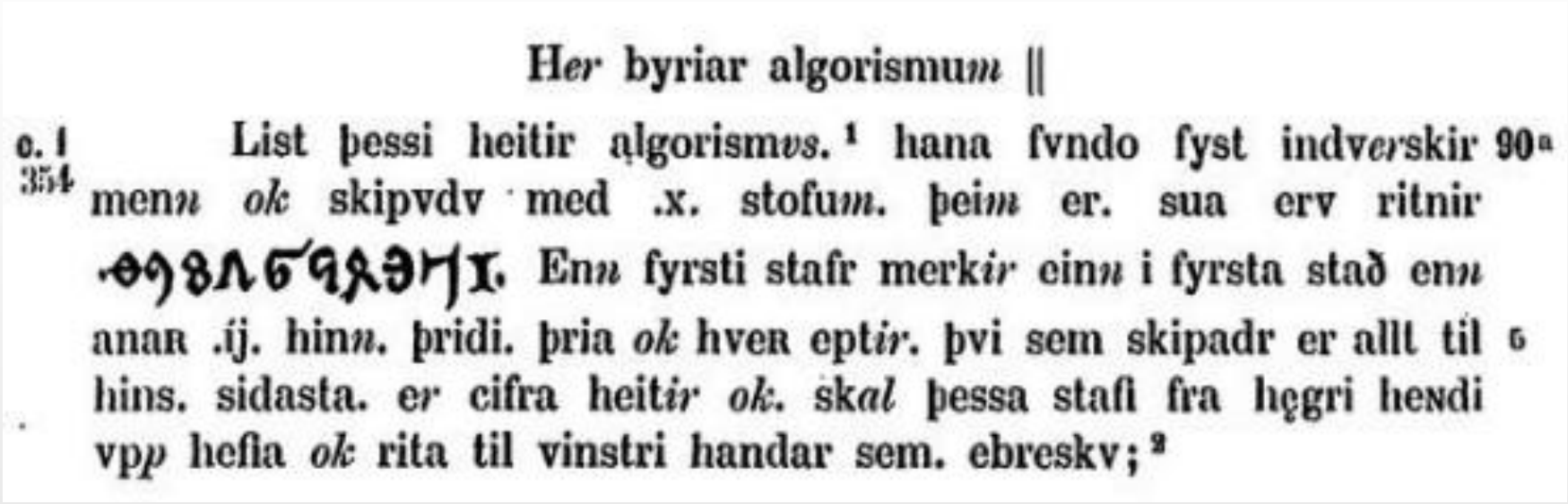
This boke is called þe boke of algorym, or Augrym after lewder
4 vse. And þis boke tretys þe Craft of Nombryng, þe quych crafte
is called also Algorym. Ther was a kyng of Inde, þe quich heyth
Algor, & he made þis craft. And after his name he called hit
algorym; or els anoþer cause is quy it is called Algorym, for þe
8 latyn word of hit s. Algorismus comes of Algos, grece, quid est
ars, latine, craft on englis, and rides, quid est numerus, latine, A
nombur on englys, inde dicitur Algorismus per addicionem huius
sillabe mus & subtraccionem d & e, quasi ars numerandi. ¶ fforther-
2 more ze most vndirstonde þat in þis craft ben vsid teen figurys,
as here bene writen for ensampul, φ 9 8 7 6 5 4 3 2 1. ¶ Expone
þe too versus afore: this present craft ys called Algorismus, in þe
quych we vse teen signys of Inde. Questio. ¶ Why ten fyguris
6 of Inde? Solucio. for as I haue sayd afore þai were fonde fyrst
in Inde of a kynge of þat Cuntre, þat was called Algor.

A derivation of Algorism.

Another derivation of the word.

Norse Translation (1310)

- Hauksbok: a Norse mathematical text derived from Villedieu's, Sacrobosco's and Fibonacci's.



Her byriar algorismum ||

o. I 354 List þessi heitir algorismvs.[1] hana fvndo fyst indverskir menn ok skipvdv med .x. stofum. þeim er. sua erv ritnir ·0123456789X. Enn fyrsti stafr merkir einn i fyrsta stað enn anar .ij. hinn. þridi. þria ok hver eptir. þvi sem skipadr er allt til hins. sidasta. er cifra heitir ok. skal þessa stafi fra hœgri hendi vpp hefia ok rita til vinstri handar sem. ebreskv;[2]

90a

5

10

The Author

Liber Chronicarum (1493)

Alexander de Villa Dei

- Uncertain biography:
  - Born in Villedieu-les-Poêles
    - When?
  - Student in Paris
    - Where?
  - Teacher in Dol, Avranches and Paris (?)
  - Friar:
    - Franciscan, Benedictine?
- Alleged works:
  - Doctrinale puerorum
  - Ecclesiale
  - Compotus ecclesiasticus
  - Algorismus
  - …

Normandy


Bréhal
Ver
Coudeville-sur-Mer
Granville
Villedieu-les-Poêles
Saint-Pair-sur-Mer
Jullouville
Carolles
Saint-Jean-le-Thomas
Saint-Laurent-de-
Brécey
Cuves
Genêts
Avranches
N175
Cancale
Saint-Malo
Dinard
Le Mont-Saint-Michel
Ducey
Isigny-le-Buat
Hirel
Saint-Jouan-des-Guérets
Beauvoir
N175
Pleurtuit
N176
Pontorson
Saint-Hilaire-du
E40
Dol-de-Bretagne
N176
Saint-Martin-de-Landelles
Saint-James
Pleudihen-sur-Rance
Epiniac
Carnet

Priest, cleric, canon, or friar?

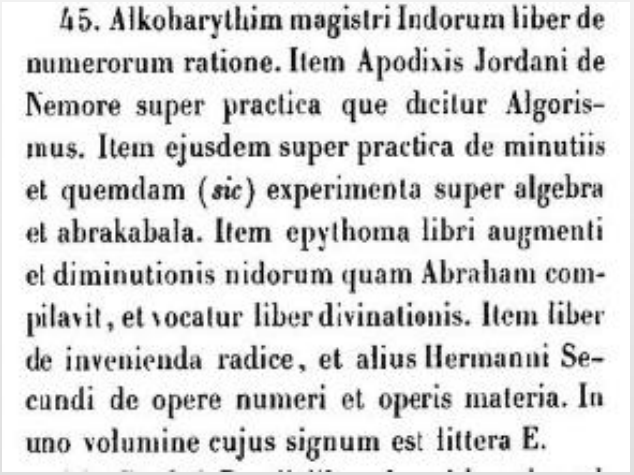
- No evidence of ordination: he is quoted as a scholar, a teacher in Dole, and a canon in Avranche
- He was born in the 1180s -> he is as old as St. Francis
  - The Franciscan rule was approved by pope Honorius III in 1223
  - The Franciscans are in France since the late 1210s
  - St. Bonaventura entered the order in 1243
  - Alexander became a Franciscan friar in his late years, according to a ms. of the XV c. (1422) now housed in Perugia, Biblioteca Comunale Lat. 112, f. 215°
    - «Auctor huis libri [Doctrinale] fuit Alexander Parigiensis (sic) cognomine de Villa Dei; cum esset senex et non potuisset amplius legere, intravit ordinem minorem et ibi mortuus fuit.»

A Franciscan Mathematician?

- According to Hughes, Franciscan professors used mathematics; Dominicans were more reluctant.
- Marianus de Florentia (1450-1523): «plurimi doctores florebant in Ordine ex quibus … Frater Alexander de Villa Dei, sacrarum litterarum professor.» (Compendium)
- It is the second source referring to Alexander as a scholar: no clues about his arithmetical works.

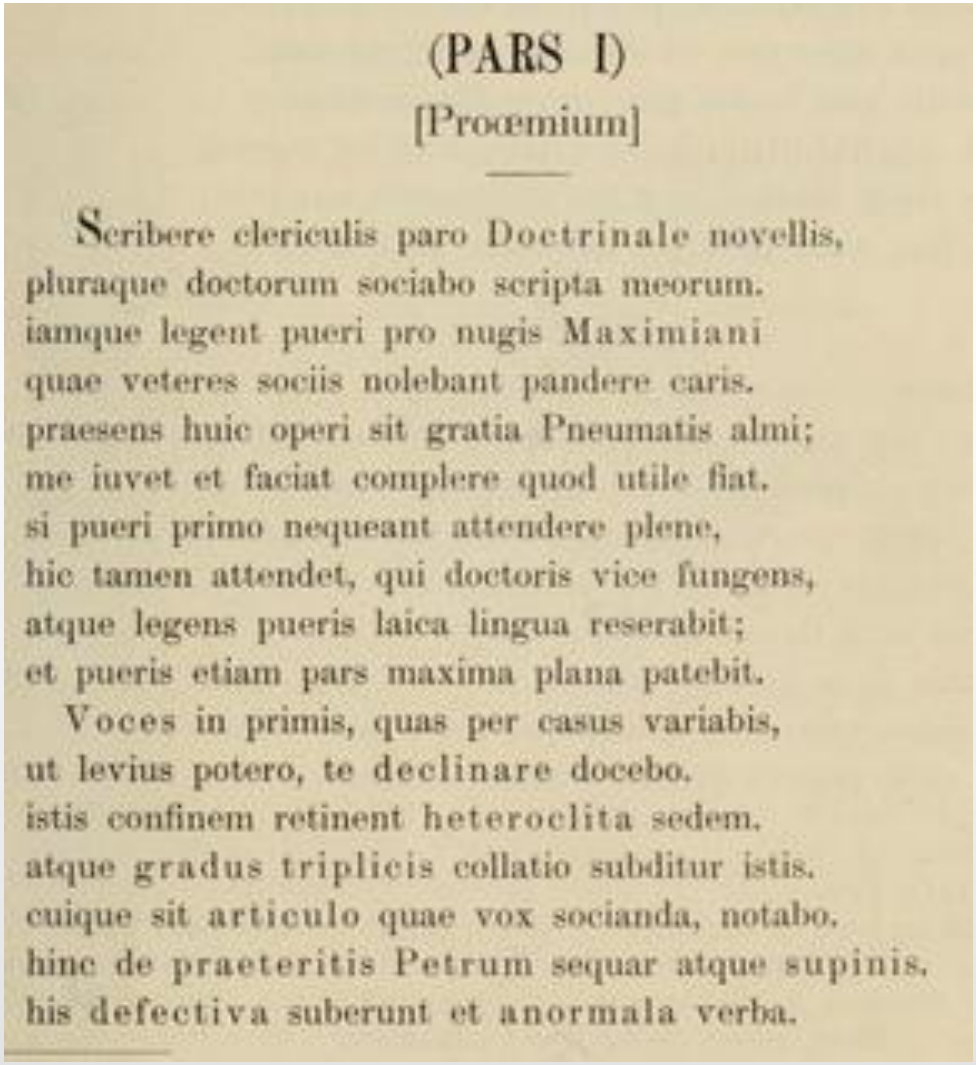
A contemporary source: Richard de Fournival

- Richard de Fournival (1201 – 1260) was a medieval philosopher and poet.
- In his Biblionomia (a list of 162 volumes, a sort of ideal library), in the shelf devoted to geometry and arithmetic, at place n. 45, "Alkoharithim magistri Indorum liber de numerorum ratione" appears, while, among grammar books, we find the Doctrinale, correctly attributed to Alexander.



45. Alkoharythim magistri Indorum liber de numerorum ratione. Item Apodixis Jordani de Nemore super practica que dicitur Algorismus. Item ejusdem super practica de minutiis et quemdam (*sic*) experimenta super algebra et abrakabala. Item epythoma libri augmenti et diminutionis nidorum quam Abraham compilavit, et vocatur liber divinationis. Item liber de invenienda radice, et alius Hermanni Secundi de opere numeri et operis materia. In uno volumine cujus signum est littera E.

Doctrinale



(PARS I)

[Proœmium]

Scribere clericulis paro Doctrinale novellis,
pluraque doctorum sociabo scripta meorum.
iamque legent pueri pro nugis Maximiani
quae veteres sociis nolebant pandere caris.
praesens huic operi sit gratia Pneumatis almi;
me iuvet et faciat complere quod utile fiat.
si pueri primo nequeant attendere plene,
hic tamen attendet, qui doctoris vice fungens,
atque legens pueris laica lingua reserabit;
et pueris etiam pars maxima plana patebit.
Voces in primis, quas per casus variabis,
ut levius potero, te declinare docebo.
istis confinem retinent heteroclita sedem.
atque gradus triplicis collatio subditur istis.
cuique sit articulo quae vox socianda, notabo.
hinc de praeteritis Petrum sequar atque supinis.
his defectiva suberunt et anormala verba.

Attribution by Halliwell

IX. *Carmen de Algorismo.* — A MS. of the Massa Compoti in the British Museu (Harl. 3902), by Alexander de Villa Dei, possesse uction to the work by some other author stated that the same author composed *et Algorismum Metricum.* M. Chasles informs me that a MS. of this tract in the French King's Library (7420. A.) has the following colophon at the end: *Explicit Algorism itus a Magistro Alexandro de Villa Dei.* This is, sufficient to prove him to be the author.

XIV c.

XIV c. Montpellier

Conclusion

- No sure evidence of authoring
  - Sources are late
  - Sacrobosco does not mention his name
- Presence of interpolation in old mss.:
  - Progression (imitating Sacrobosco?), multiplication
- Presence of other works in verse about scientifical content.