Impact of Informatics on Mathematics and its Teaching

On the Importance of Epistemological Analysis to Feed Didactical Research

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The teaching of mathematics has been questioned for more than 30 years by the development of *computer science* (*informatics* in the following) due to its strong relation with mathematics [1], [2]. Today, we witness the generalization of the teaching of informatics (inside or beside mathematics), the introduction of contents shared with informatics in mathematics curricula (like algorithmics), and the generalization of computers as tools for teaching, especially in mathematics. Naturally, those changes raise many educational questions that have been studied a lot. Although, we want to focus on some of these questions with the point of view of epistemology and didactics. We will exemplify these questions and show how important is the epistemology of the informatics-mathematics relation in order to tackle these issues.

Indeed, mathematics and informatics have strong links and a common history. More precisely, (1) they share common foundations, structured by logics, and a specific relation with proof [3], (2) there is a certain continuity between them with many fields developing at their interface, (3) Computer assisted mathematics change the way some mathematicians work, involving an experimental dimension [4], and (4) mathematics and informatics, sometimes classified as formal sciences, have a very similar relation to other sciences (this will not be discussed here).

Didactics of sciences, in the French tradition, and particularly didactics of mathematics, have historically a strong and fundamental relation to epistemology. Artigue [5] shows how the epistemological analysis, by enlightening the specificity of mathematical activity, the way concepts appear and develop, and phenomena like didactical transposition, contribute to what she calls *epistemological vigilance in didactics*, and permits to understand errors and obstacles and to organize the teaching and learning of mathematics.

This approach can be extended to the mathematics-informatics interactions, and particularly the impacts of informatics on mathematics and its practice. This is what we want to exemplify and defend here.

Proof and algorithm. The close links between proof and algorithm, as the theoretical result that any proof can be seen as an algorithm and any algorithm as a proof, the theories of algorithms proving, or the fact that algorithms can find or certificate proofs, must certainly question teaching and learning of mathematics. Algorithms could be questioned as a mean for a mediation in the teaching of mathematical proof, particularly in a context of presence of algorithmics in a mathematical curriculum [6].

Language. Different types of variables in mathematics and informatics can be distinguished. Didactics of mathematics have already deeply analysed and documented the obstacles caused by the introduction of variables in elementary algebra. The introduction of elementary algebra simultaneously with algorithmics or programming and the associated notion of variable (sometimes even used as a mean to introduce mathematical variables) must be questioned through the epistemological and didactical lens.

Algorithmic thinking and mathematical thinking. These thinkings share much but have fundamental differences [7], [8], especially when asking what is a (good) answer, with an emphasis on complexity and efficiency in informatics : a mathematical simplification of a formula is not always easier to compute than the original formula [9], [10] (e.g. in combinatorics), and can not be considered as an algorithmic answer.

Experimental mathematics and role of the computer. Informatics allowed to develop or renew experimental aspects of mathematics [4], [11]. Epistemology of mathematics can be considered as unchanged with computer-assisted mathematics, but works like [12], [13] show that changes are deep enough to be taken into account in educational issues [14], [15] and it is important to study the way it is transposed to the teaching of mathematics.

New objects. The development of informatics brought new objects in mathematics, mainly of discrete type, the were sometimes present but not considered seriously before informatics. Introducing computer in the classroom inevitably leads to mathematical questions about such objects (explicitly presented or not) like *representations of numbers* in a machine when programming or *discrete lines* when using a dynamic geometry software or plotting curves. This directly questions the consistency of the mathematics curricula and the necessity of questioning those curricula regarding new fields arising in mathematics.

In conclusion, we defend the importance to consider the epistemology of informatics in the didactics of mathematics. It seems important to take into account the way concepts in informatics and mathematics arose, the links informatics had and have with mathematics and also the specificities that distinguished it from mathematics, the role of logics and language and the place of proof. Through our examples we underlined two big lines that must be studied: (1) the relations between proof, languages, algorithm and logic, and (2) the new fields appearing at the mathematics-informatics interface, discrete mathematics and their relation to continuous. The examples presented open perspectives in this direction.

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